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MATHEMATICS

Coefficient Identification Problems for Semilinear Parabolic Equations

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For a semilinear parabolic equation with a sufficiently general nonlinearity, we consider the problem of identifying the coefficients multiplying the nonlinear term and the time derivative and the problem of identifying the source function and the coefficient multiplying the nonlinear term. In the case of Cauchy data, the unique classical solvability of these problems is proved locally in the class of sufficiently smooth bounded functions (together with the corresponding derivatives) [1, 2]. For the problem of identifying one coefficient in the case of an equation with a quadratic nonlinearity, solvability in the class of functions vanishing at infinity with respect to a single distinguished variable was investigated in [3]. Some identification problems for two coefficients can be found in [4–7].

IDENTIFICATION OF THE COEFFICIENTS MULTIPLYING THE TIME DERIVATIVE AND THE NONLINEAR TERM

In the domain $G_{[0,T]} = \{(t, x, z)| \, 0 \le t \le T, x \in E_1, z \in E_1\}$, we consider the Cauchy problem

$$\lambda_1(t,x)u_t = a_1(t,x)u_{xx} + a_2(t,x)u_{zz} + \lambda_2(t,x)M(t,u) + f(t,x,z), \qquad (1)$$

$$u(0,x,z) = u_0(x,z), \quad x \in E_1, \quad z \in E_1. \qquad (2)$$

The functions $a_1(t, x)$ and $a_2(t, x)$ are assumed to be strictly positive: $a_1(t, x) \ge \alpha > 0$ and $a_2(t, x) \ge \alpha > 0$ for $(t, x) \in \Pi_{[0,T]} = \{(t, z)| \, 0 \le t \le T, x \in E_1\}$, where $\alpha =$ const. The functions $M(t, y)$, $u_0(x, z)$, and $f(t, x, z)$ are real-valued and defined on $[0, T] \times E_1$, E_2, and $G_{[0,T]}$, respectively. Here and below, E_n denotes a Euclidean space of dimension n.

The functions $\lambda_1(t, x)$ and $\lambda_2(t, x)$ are to be determined together with the solution $u(t, x, z)$ to problem (1), (2) that obeys the overdetermination conditions

$$u(t,x,0) = \varphi_1(t,x), \quad u_z(t,x,0) = \varphi_2(t,x) \qquad (3)$$

and the matching conditions

$$u_0(x,0) = \varphi_1(0,x), \quad \frac{\partial}{\partial z}u_0(x,0) = \varphi_2(0,x). \qquad (4)$$

Assume that $M(t, y)$ is sufficiently smooth (i.e., all of its derivatives in (5) are continuous) and

$$\left|\frac{\partial^j}{\partial y^j}M(t,y)\right| \le M_0(1+|y|^p), \qquad (5)$$
$$j = 0, 1, \ldots, 11, \quad 0 \le t \le T, \quad y \in E_1.$$

Here, M_0 is a constant; $p \ge 1$ is an integer; $M^{(j)}(t, y) = \frac{\partial^j}{\partial y^j}M(t, y)$, where $j \ge 1$ is an integer; and $M^{(0)}(t, y) = M(t, y)$. Let

$$|\Delta(t,x)| \equiv \left|\frac{\partial}{\partial t}\varphi_1(t,x)M^{(1)}(t,\varphi_1)\varphi_2(t,x) - \frac{\partial}{\partial t}\varphi_2(t,x)M(t,\varphi_1)\right| \ge \delta > 0 \qquad (6)$$

for $(t, x) \in \Pi_{[0,T]}$. Here, δ is a fixed constant.

It is assumed that the input data are sufficiently smooth (i.e., all the derivatives in (7) are continuous) and

$$\left|\frac{\partial^m}{\partial x^m}\psi_i(t,x)\right| + \left|\frac{\partial^m}{\partial x^m}a_i(t,x)\right| + \left|\frac{\partial^{m+1}}{\partial x^m \partial t}\varphi_i(t,x)\right| + \left|\frac{\partial^k}{\partial z^k}\frac{\partial^m}{\partial x^m}u_0(x,z)\right| + \left|\frac{\partial^k}{\partial z^k}\frac{\partial^m}{\partial x^m}f(t,x,z)\right| \le C, \qquad (7)$$
$$m = 0, 1, 2, 3, 4,$$
$$k = 0, 1, \ldots, 11-2m, \quad i = 1, 2, \quad (t,x,z) \in G_{[0,T]},$$

where C is a nonnegative constant. Here and below, C denotes generally various nonnegative constants.

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Assume that

$$A_1(t,x) + A_2(t,x)\frac{\partial^2}{\partial z^2}u_0(x,0) + A_3(t,x)\frac{\partial^3}{\partial z^3}u_0(x,0) \ge \delta \qquad (8)$$

for $(t, x) \in \Pi_{[0, T]}$. Here,

$$A_1(t,x) = \frac{\psi_1(t,x)M^{(1)}(t,\varphi_1(t,x))\varphi_2(t,x) - \psi_2(t,x)M(t,\varphi_1(t,x))}{\Delta(t,x)},$$

$$A_2(t,x) = \frac{a_2(t,x)M^{(1)}(t,\varphi_1(t,x))\varphi_2(t,x)}{\Delta(t,x)},$$

$$A_3(t,x) = -\frac{a_2(t,x)M(t,\varphi_1(t,x))}{\Delta(t,x)},$$

$$\psi_1(t,x) = a_1(t,x)(\varphi_1)_{xx} + f(t,x,0),$$

$$\psi_2(t,x) = a_1(t,x)(\varphi_2)_{xx} + f_z(t,x,0).$$

Theorem 1. *Let conditions* (4)–(8) *be satisfied.*

Then, *problem* (1)–(3) *has a solution* $u(t, x, z)$, $\lambda_1(t, x)$, $\lambda_2(t, x)$ *in the class*

$$Z(t^*) = \left\{u, \lambda_1, \lambda_2 \,\middle|\, u_t, \frac{\partial^{k+l}}{\partial z^k \partial x^l}u, \frac{\partial^{k+1}}{\partial z^{k+1}}u, \frac{\partial^{k+2}}{\partial z^{k+2}}u \in C(G_{[0,t^*]}), \lambda_1(t,x), \lambda_2(t,x) \in C^{0,2}_{t,x}(\Pi_{[0,t^*]}); \; k = 0, 1, 2, 3; \quad l = 0, 1, 2\right\},$$

and that solution satisfies the relations

$$\sum_{k=0}^{5}\left|\frac{\partial^k}{\partial z^k}u(t,x,z)\right| + \sum_{k=0}^{3}\sum_{m=0}^{2}\left|\frac{\partial^k}{\partial z^k}\frac{\partial^m}{\partial x^m}u(t,x,z)\right| \le C, \quad (t,x,z) \in G_{[0,t^*]}, \qquad (9)$$

$$\sum_{m=0}^{2}\left|\frac{\partial^m}{\partial x^m}\lambda_1(t,x)\right| + \sum_{m=0}^{2}\left|\frac{\partial^m}{\partial x^m}\lambda_2(t,x)\right| \le C, \quad (t,x) \in \Pi_{[0,t^*]}. \qquad (10)$$

Here, t^* *is a constant depending on* δ *and the constants* C *in* (7), $0 < t^* \le T$, *and*

$$C^{0,2}_{t,x}(\Pi_{[0,t^*]}) = \left\{g(t,x) \,\middle|\, \frac{\partial^m}{\partial x^m}g(t,x) \in C(\Pi_{[0,t^*]}), m = 0, 1, 2\right\}.$$

Theorem 2. *The solution* $u(t, x, z)$, $\lambda_1(t, x)$, $\lambda_2(t, x)$ *to problem* (1)–(6) *that satisfies relations* (9) *and* (10) *is unique in the class* $Z(t^*)$.

Theorems 1 and 2 imply the following result.

Theorem 3. *Let conditions* (4)–(8) *be satisfied.*

Then, *problem* (1)–(3) *has a unique solution* $u(t, x, z)$, $\lambda_1(t, x)$, $\lambda_2(t, x)$ *in* $Z(t^*)$ *that satisfies* (9) *and* (10).

IDENTIFICATION OF THE SOURCE FUNCTION AND THE COEFFICIENT OF THE NONLINEAR TERM

In the domain $G_{[0,T]} = \{(t, x, z) | 0 \le t \le T, x \in E_n, z \in E_1\}$, we consider the Cauchy problem

$$u_t(t,x,z) = L_x(u) + u_{zz} + \lambda_1(t,x)M(t,u(t,x,z)) + \lambda_2(t,x)f(t,x,z), \qquad (11)$$

$$u(0,x,z) = u_0(x,z), \quad x \in E_n, \quad z \in E_1. \qquad (12)$$

Here, $L_x(u) = \sum_{k,m=1}^{n} a_{km}(t)u_{x_k x_m} + \sum_{k=1}^{n} a_k(t)u_{x_k}$, where $a_{km}(t)$, $a_k(t) \in C[0, T]$.

The functions $M(t, y)$, $u_0(x, z)$, and $f(t, x, z)$ are real-valued and defined on $[0, T] \times E_1$, E_2, and $G_{[0,T]}$, respectively.

The functions $\lambda_1(t, x)$ and $\lambda_2(t, x)$ are to be determined together with the solution $u(t, x, z)$ to problem (11), (12) that obeys overdetermination conditions (3) and matching conditions (4).

Assume that $M(t, y)$ is sufficiently smooth (i.e., all of its derivatives in relation (13) below are continuous) and

$$\left|\frac{\partial^j}{\partial y^j}M(t,y)\right| \le M_0(1 + |y|^p), \quad j = 0, 1, \ldots, 9, \quad 0 \le t \le T, \quad y \in E_1. \qquad (13)$$

Let

$$|M(t,\varphi_1(t,x))f_z(t,x,0) - M^{(1)}(t,\varphi_1(t,x))\varphi_2(t,x)f(t,x,0)| \ge \delta > 0 \qquad (14)$$

for $(t, x) \in \Pi_{[0,T]} = \{(t, x) | 0 \le t \le T, x \in E_n\}$. Here, δ is a fixed constant.

It is assumed that the input data are sufficiently smooth (i.e., all the derivatives in relations (15) below are continuous) and

$$|D_x^\beta \psi_i(t,x)| + \left|\frac{\partial^k}{\partial z^k}D_x^\beta u_0(x,z)\right| + \left|\frac{\partial^k}{\partial z^k}D_x^\beta f(t,x,z)\right| \le C, \qquad (15)$$

$$|\beta| = m, \quad m = 0, 1, 2, 3, 4, \quad k = 0, 1, \dots, 5,$$
$$(t, x, z) \in G_{[0, T]},$$

where C is a positive constant. Here,

$$\psi_1(t, x) = (\varphi_1)_t - L_x(\varphi_1) - f(t, x, 0),$$

$$\psi_2(t, x) = (\varphi_2)_t - L_x(\varphi_2) - f_z(t, x, 0).$$

The following theorems hold.

Theorem 4. *Let conditions* (3), (4), *and* (13)–(15) *be fulfilled.*

Then, problem (11), (12), (3) *has a solution* $u(t, x, z)$, $\lambda_1(t, x)$, $\lambda_2(t, x)$ *in the class*

$$Z(t^*) = \left\{u, \lambda_1, \lambda_2 \middle| u_t, \frac{\partial^k}{\partial z^k} D_x^\alpha u \in C(G_{[0, t^*]}),\right.$$

$$\lambda_1(t, x), \lambda_2(t, x) \in C_{t, x}^{0, 2}(\Pi_{[0, t^*]});$$

$$\left. k = 0, 1, 2, 3; \quad |\alpha| \le 2 \right\},$$

and that solution satisfies the relations

$$\sum_{|\alpha| \le 2} \sum_{k=0}^{3} \left| D_x^\alpha \frac{\partial^k}{\partial z^k} u(t, x, z) \right| \le C, \tag{16}$$

$$\sum_{|\alpha| \le 2} \left| D_x^\alpha \lambda_1(t, x) \right| + \sum_{|\alpha| \le 2} \left| D_x^\alpha \lambda_2(t, x) \right| \le C \tag{17}$$

for $(t, x, z) \in G_{[0, t^*]}$. *Here, t^* is a constant depending on* δ *and the constants C in* (15), $0 < t^* \le T$, *and*

$$C_{t, x}^{0, 2}(\Pi_{[0, t^*]})$$
$$= \{g(t, x) | D_x^\alpha g(t, x) \in C(\Pi_{[0, t^*]}), |\alpha| \le 2\}.$$

Theorem 5. *The solution* $u(t, x, z)$, $\lambda_1(t, x)$, $\lambda_2(t, x)$ *to problem* (11), (12), (3) *that satisfies relations* (16) *and* (17) *is unique in the class* $Z(t^*)$.

Theorem 6. *Let conditions* (3), (4), *and* (13)–(15) *be fulfilled.*

Then, problem (11), (12), (3) *has a unique solution* $u(t, x, z)$, $\lambda_1(t, x)$, $\lambda_2(t, x)$ *in the class* $Z(t^*)$ *that satisfies relations* (6) *and* (7).

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